THE RELATIONSHIP BETWEEN THE LOADING LEVEL AND CAPACITY OF THE SEA PORT TERMINAL

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The necessity to determine the optimal loading level of a sea port and its terminals arises when the optimal structure of the park of cargo handling equipment is being investigated [1]. Consider the terminal which consists of interchangeable berths equipped with facilities of the same type. So any unused berth is available for the cargo operations when ship arrives.

The capacity of the sea cargo front (SCF) of terminal is one of the most important characteristics which determines capacity of the terminal in whole. By capacity of SCF we mean the maximal amount of cargo that can be processed over a given time period (usually a year) with the fullest use of the equipment and calendar operational time. In accordance with [2], daily capacity of SCF which consists of berths can be determined as

\[ \Pi = \sum_{i=1}^{m} P_i, \quad P_i = \frac{Q_{ci} t_{cf}}{Q_{ci}/N_i + P_{lie} + t_{aux}}, \]

where \( P_i \) denotes daily capacity of SCF of \( i \)-th berth, \( Q_{ci} \) denotes the vessel capacity, \( t_{cf} \) is the average time period when berth is busy during a day, \( N_i \) is the number of technological lines on the SCF; \( P_{lie} \) is the performance of a technological line, \( t_{aux} \) is the duration of auxiliary operations performed with the ship at the berth, including the waiting time.

The loading level of SCF \( Q \) should not exceed its capacity, otherwise it will be impossible for the terminal to cope with the cargo traffic. The situation, when the loading level matches SCF capacity is acceptable. In this case the roads will be empty only if vessels will arrive strictly in time with intervals which equals their processing time. However, the irregularities in time of ships arriving and their processing time which, in fact, cannot be eliminated, cause a large number of vessels in the roads when loading level of SCF approaches its capacity, which in turn cause significant financial losses for port clients. That is why the loading level should be restricted by the appropriate value which is noticeably less then capacity of SCF. In accordance with specifications [2], the suggested reasonable loading level of SCF should be found as

\[ Q = k_z \Pi, \]

<table>
<thead>
<tr>
<th>Malaksiano O.A., Malaksiano M.O. Співвідношення завантаження і пропускної спроможності терміналу морського порту.</th>
<th>Малаксіано О.А., Малаксіано М.О. Співвідношення завантаження і пропускної спроможності терміналу морського порту.</th>
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<tbody>
<tr>
<td>Досліджується взаємозв'язок між рівнем завантаження терміналу морського порту і показниками обслуговування суден. Запропоновано математичну модель, яка заснована на застосуванні теорії масового обслуговування. Отримані результати можуть бути використані для знаходження оптимального рівня завантаження терміналу.</td>
<td>Малаксіано А.А., Малаксіано Н.А., Соотношені загрузки и пропускной способности терминала морского порта. Исследуется взаимосвязь между уровнем загрузки терминала морского порта и показателями обслуживания судов. Предложена математическая модель, основанная на применении теории массового обслуживания. Полученные результаты могут быть использованы для нахождения оптимального уровня загрузки терминала.</td>
</tr>
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<td>Ключові слова: показники обробки суден, рівень завантаження комплексу</td>
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\[ T \]
where $k_2$ is the employment ratio, which characterizes the rate of employment of berths. Despite the practical importance, however, no significant results have been proposed to estimate the value of $k_2$ so far. The suggested value of employment ratio should be chosen from 0.6 to 0.7 for universal terminals [2], from 0.5 to 0.6 for specialized bulk or timber terminals, and from 0.4 to 0.5 for container, ro-ro or oil terminals. This specifications does not take into account such important factors as vessels traffic intensity and cargo handling operation technology. Very often prescribed in this way employment ratio can considerably vary from its optimal. For more precise estimations of employment ratio different authors propose evaluations based on empirical exploration of relation between vessels processing indicators and SCF loading level under different circumstances. This approach has obvious disadvantages which considerably complicate its implementation in practice.

The purpose of the article

The aim of the article is to investigate the relationship between the vessels processing indicators and the loading level of the terminal by means of the appropriate mathematical model based on the application of queueing theory.

The main material

Consider a terminal which consists of berths. Despite the attempts to draw up and follow schedules, very often the vessels arrival dates and processing times are subject to various arbitrary factors. Denote $P_j(t)$ the probability that the number of busy berths in the moment of time $t$ equals $j$. It is obvious, that

$$\sum_{j=0}^{\infty} P_j = 1.$$  
If the vessel arrives and several berths are not busy, we will assume that the vessel will occupy the berth with the highest capacity. Also assume that every berth can receive only one vessel at the same time and all available resources are to be shared equally between vessels which occupy the same straight-line area of the terminal. For some reasons it might be interesting to consider another rules of resources allocations and different queue regulations. Although the following model is also applicable for various cases, they are not considered here.

The ability of stationary machines to be displaced from one berth to another depends first of all on berthing configuration of the terminal. Consider the simplest case, when terminal has broken-line shaped berthage, which does not permit stationary machines to be displaced to another berths, and berths have different capacities. Let $t_j$ be the average vessel processing time for the $j$-th berth,

$$t_j = \frac{Q_j \cdot 24}{N_j P_{j} \cdot t_{ef}} + \tau_{aou}, \quad j = 1, m,$$

Then if $\gamma$ of berths are occupied then vessels processing intensity equals

$$\mu_\gamma = \sum_{j=1}^{\gamma} \frac{1}{t_j}.$$  

If vessel processing times are the same for all berths $t_1 = t_2 = \text{const}$, then $\mu_\gamma = \gamma \frac{1}{t_1}$. If terminal has a straight-line shaped berthage, then stationary equipment can be displaced by means of crane tracks from a vacant berth to adjacent occupied one. In this case the intensity of cargo handling operation when $\gamma$ of berths are occupied equals

$$\mu_\gamma = \begin{cases} \frac{1}{t(K)}, & \text{if } \gamma K \leq N, \\ \frac{1}{t(N/\gamma)}, & \text{if } \gamma K > N, \end{cases}$$

where $K$ is the upper bound of mooring machines concentration on a vessel [3], $N$ is the general number of machines at the terminal, $t(n)$ is the mean processing time when vessel is processed by $n$ machines.

Now consider the influence of vessels traffic intensity on the mean vessels processing time, waiting time in the roads and the other indicators, provided all other conditions and parameters of the terminal stay unchanged. It is obvious that the simple birth and death scheme is not applicable for this case. That is why for studying this problem we will proceed from the general model of Markovian chain. Assume that the time between vessels arrival and vessels processing time are random variables with the Poisson distribution. Consider a terminal which consists of three berths and restricted road with upper bound of vessels. Under the circumstances, the terminal can be modeled as a stochastic system each state of which can be defined by the set of four variables $(i, j, k, r)$, where $i$, $j$ and $k$ are equal to 1 if respectively the first, second and third berths are busy, and equal to 0 if appropriate berths are vacant. Value of the last variable $r$ indicates the number of ships in the roads, $r = 0, ..., n$. For convenience denote:

- $C_1 = (0,0,0,0)$, $C_2 = (1,0,0,0)$, $C_3 = (0,1,0,0)$,
- $C_4 = (0,0,1,0)$, $C_5 = (1,0,0,0)$, $C_6 = (0,1,0,0)$,
- $C_7 = (0,1,1,0)$, $C_8 = (1,1,0,0)$, $C_9 = (1,1,0,0)$,
- $C_{10} = (1,1,1,0)$, $C_{11} = (1,1,1,2)$, $C_{8n} = (1,1,1, n)$

For the referred above model, the intensity of vessels processing on the $i$-th berth in the moment of time $t$ depends on state of other berths at the moment. Let $\mu_i$ be the mean intensity of vessels processing on the $i$-th berth, when terminal is in the $\gamma$-th state, and $\lambda$ is the mean intensity of vessels arriving. Assume that $\mu_\gamma = \mu_\bar{\gamma}$ for all $i = \overline{1,3}$, $\gamma = 9, n$. Denote $\mu = \mu_\bar{\gamma} + \mu_2 \gamma + \mu_3 \gamma$. Consider random events which consist in finding the terminal
in the state $C_i$, $i = 1, \ldots, 8 + n$ at the fixed moment of time $t$. Denote $p_i(t)$ the probability to find the terminal in the state $C_i$ at the moment of time $t$. If all probabilities $p_i(t)$ are known, one can easily obtain all required vessels processing indicators. In order to find these probabilities using well-known method [4] we will reduce the problem to study of appropriate differential equation. Fix an arbitrary moment of time and small time interval $\Delta t$. The probability that at the moment of time $t + \Delta t$ the terminal will be in the state $C_1$ equals

$$p_1(t + \Delta t) = p_1(t) \cdot (1 - \lambda \cdot t) + p_1(t) \cdot \mu_{12} \cdot t + p_1(t) \cdot \mu_{23} \cdot \Delta t + p_4(t) \cdot \mu_{44} \cdot \Delta t + o(\Delta t).$$

(1)

The first summand in the right part of (1) expresses the probability of situation when there were no vessel at the time at the terminal and none has arrived during the interval after. The sum of the rest of summands express the probability that there were one vessel at the moment of time at the terminal and by the moment of time it will be processed. After the passage to the limit when from the equation (1) follows

$$p_1(t) = p_1(t) \cdot (1 - \lambda \cdot t) + p_1(t) \cdot \mu_{12} + p_2(t) \cdot \mu_{23} + p_4(t) \cdot \mu_{44}$$

In much the same way one can obtain equations for the rest of states of the terminal. As the result we obtain the following system of differential equations.

$$\frac{dp_1(t)}{dt} = -p_1(t) \cdot \lambda + p_1(t) \cdot \mu_{12} + p_2(t) \cdot \mu_{23} + p_4(t) \cdot \mu_{44}$$

$$\frac{dp_2(t)}{dt} = p_1(t) \cdot (\lambda \cdot \mu_{12}) + p_2(t) \cdot (\lambda \cdot \mu_{23}) + p_3(t) \cdot \mu_{32}$$

$$\frac{dp_3(t)}{dt} = -p_2(t) \cdot \mu_{23} + p_3(t) \cdot (\lambda \cdot \mu_{32}) + p_4(t) \cdot \mu_{43}$$

$$\frac{dp_4(t)}{dt} = -p_3(t) \cdot \mu_{32} + p_4(t) \cdot \mu_{44}$$

(2)

The solution of system (2) provided normalization condition $\sum_{i=1}^{8} p_i = 1$ and appropriate initial conditions gives a full information about the queuing system. By means of passage to the limit when $t \to \infty$ system of differential equations (2) transforms into the following system of linear equations (3), where $p_i = \lim_{t \to \infty} p_i(t)$ expresses the limiting probabilities when the terminal runs in the stationary mode.

$$0 = -p_1 \cdot \lambda + p_1 \cdot \mu_{12} + p_2 \cdot \mu_{23} + p_4 \cdot \mu_{44}$$

$$0 = p_1 \cdot (\lambda \cdot \mu_{12}) + p_2 \cdot (\lambda \cdot \mu_{23}) + p_3 \cdot \mu_{32}$$

$$0 = -p_2 \cdot \mu_{23} + p_3 \cdot (\lambda \cdot \mu_{32}) + p_4 \cdot \mu_{43}$$

$$0 = -p_3 \cdot \mu_{32} + p_4 \cdot \mu_{44}$$

(3)

Now when solution of (3) is obtained, the vessels processing indicators can be easily calculated.

In accordance with the established order, the capacity of a terminal is calculated over a long period of time, usually longer than year allowing for meteorological factors and necessary equipment service. In accordance with specifications [5], we will distinguish the following types of repair works: maintenance, scheduled repair (routine repair, capital repair) and emergency repair. Maintenance is usually carried out during the reception and delivery of shifts, or in the intervals of cargo operations. Rarely for this purpose, machines can be removed from service, but usually no more than for one or two shifts a month. So maintenances do not affect noticeably the capacity of the terminal. Frequency and duration of the scheduled repairs are governed by existing specifications [5], and their implementation is controlled by appropriate state supervisory authorities. The duration of one routine repair and capital repair equals approximately to one and three months respectively. The frequencies of such repair works are determined by operating time. For example the scheduled repairs of portal cranes at the average are carried out once in two or three years. The capital repair is carried out once for every five routine repairs. During its life cycle portal crane for example, usually undergoes one or two capital repairs. The economically substantiated terms of repairs and retirement for terminal port equipment when forecast level of employment is subject to change can be determined by [6]. The scheduled repairs take a lot of time and that is why they should be taken into account when capacity of the terminal is estimated. Denote $T_f$ the forecasting time-frame. Let $T_L$ be the total time when $L$ machines are under repair. It is obvious that

$$\sum_{L=0}^{N} T_L = T,$$

where $N$ is general number of machines at the terminal. The values of $L$ and $T_L$ can be easily found from the preventive overhaul schedule. Besides scheduled repairs, machines can be put out of action for long terms because of emergency repairs. The emergency breakdown of equipment is a random event and the duration of its repair is also a random variable. That is why the number of machines $A_L$ being at the same time under emergency repair is a random variable. The mean number of up state machines at the terminal $N_L$ should be found.
allowing for number and characteristics of equipment at the terminal and number of available service stations [7]. And the capacity of the area of a sea cargo front \( I(N_t) \) is based on the number of the good state machines. The mean capacity of the whole sea cargo front on the forecasting time-frame \( T \), allowing for the interruptions caused by meteorological reasons and repairs can be estimated by the formula

\[
I_T = \sum_{L=0}^{n} T_L k_{md} I(N_L)
\]

where \( k_{md} \) is the coefficient of outage caused by meteorological conditions [2].

Now for illustration consider the particular case when the number of vessels in the roads is bounded by 12, terminal has a straight-line berthing and consists of three berths equipped by nine rigs with performance of 50 tons per hour each. Assume the mean vessel capacity of 10000 tons. And put the upper bound of concentration equals three rigs per vessel. In this case the mean ships processing conditional intensities are

\[
\mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5 = \mu_6 = \mu_7 = \mu_8 = 0,0153846
\]

The analysis of this example shows that the gradual increase in the loading level leads to the moderate increase in the mean vessels processing time from 65 up to 80 hours, while the mean waiting time in the roads at the same time undergoes the considerable growth from 1 up to 132 hours. The vessels processing indicators are almost independent from the loading level of the terminal if its value does not exceed 0.38, and are subjected to considerable change for the worse otherwise (table 1, picture 1).

### Table 1. The relationship between the vessels processing indicators and the loading level of the terminal

<table>
<thead>
<tr>
<th>Loading level of the terminal</th>
<th>Vessels arriving intensity, vessels per hour</th>
<th>The probability to find no vessels at the terminal</th>
<th>The probability to find one vessel at the terminal</th>
<th>The probability to find two vessels at the terminal</th>
<th>The probability to find three vessels at the terminal</th>
<th>The mean number of vessels at the terminal without the road</th>
<th>The mean cargo handling processing time, hours</th>
<th>The mean waiting time in the roads, hours</th>
<th>The mean overall waiting time (time in the roads + handling processing time), hours</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>0.002</td>
<td>0.878</td>
<td>0.114</td>
<td>0.007</td>
<td>0.114</td>
<td>0.000</td>
<td>0.000</td>
<td>0.013</td>
<td>65.126</td>
</tr>
<tr>
<td>0.11</td>
<td>0.004</td>
<td>0.770</td>
<td>0.200</td>
<td>0.026</td>
<td>0.200</td>
<td>0.000</td>
<td>0.000</td>
<td>0.026</td>
<td>65.467</td>
</tr>
<tr>
<td>0.16</td>
<td>0.006</td>
<td>0.675</td>
<td>0.263</td>
<td>0.051</td>
<td>0.263</td>
<td>0.002</td>
<td>0.000</td>
<td>0.039</td>
<td>65.796</td>
</tr>
<tr>
<td>0.22</td>
<td>0.008</td>
<td>0.591</td>
<td>0.307</td>
<td>0.080</td>
<td>0.307</td>
<td>0.005</td>
<td>0.000</td>
<td>0.054</td>
<td>66.619</td>
</tr>
<tr>
<td>0.27</td>
<td>0.010</td>
<td>0.515</td>
<td>0.335</td>
<td>0.109</td>
<td>0.335</td>
<td>0.011</td>
<td>0.000</td>
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<td>0.349</td>
<td>0.136</td>
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<td>0.022</td>
<td>0.000</td>
<td>0.082</td>
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<tr>
<td>0.38</td>
<td>0.014</td>
<td>0.388</td>
<td>0.353</td>
<td>0.161</td>
<td>0.353</td>
<td>0.038</td>
<td>0.000</td>
<td>0.097</td>
<td>69.122</td>
</tr>
<tr>
<td>0.44</td>
<td>0.016</td>
<td>0.334</td>
<td>0.347</td>
<td>0.180</td>
<td>0.347</td>
<td>0.061</td>
<td>0.001</td>
<td>0.112</td>
<td>70.093</td>
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<tr>
<td>0.49</td>
<td>0.018</td>
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<td>0.333</td>
<td>0.195</td>
<td>0.333</td>
<td>0.092</td>
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<td>71.112</td>
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<td>0.54</td>
<td>0.020</td>
<td>0.241</td>
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<td>0.203</td>
<td>0.313</td>
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<tr>
<td>0.60</td>
<td>0.022</td>
<td>0.201</td>
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<td>0.205</td>
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<tr>
<td>0.65</td>
<td>0.024</td>
<td>0.165</td>
<td>0.257</td>
<td>0.201</td>
<td>0.257</td>
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<td>0.71</td>
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<td>0.133</td>
<td>0.224</td>
<td>0.189</td>
<td>0.224</td>
<td>0.320</td>
<td>0.024</td>
<td>1.964</td>
<td>75.464</td>
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<tr>
<td>0.76</td>
<td>0.028</td>
<td>0.104</td>
<td>0.189</td>
<td>0.172</td>
<td>0.189</td>
<td>0.404</td>
<td>0.034</td>
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<tr>
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<td>0.030</td>
<td>0.079</td>
<td>0.153</td>
<td>0.150</td>
<td>0.153</td>
<td>0.496</td>
<td>0.044</td>
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<td>0.87</td>
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<td>0.119</td>
<td>0.124</td>
<td>0.119</td>
<td>0.591</td>
<td>0.054</td>
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<td>0.040</td>
<td>0.089</td>
<td>0.098</td>
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<td>0.682</td>
<td>0.062</td>
<td>2.604</td>
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<tr>
<td>0.98</td>
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<td>0.027</td>
<td>0.063</td>
<td>0.074</td>
<td>0.063</td>
<td>0.764</td>
<td>0.065</td>
<td>2.719</td>
<td>80.001</td>
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</table>
This picture allows to analyze the relationship between the average vessels processing indicators and the loading level of the terminal. Although the average values of these indicators may be satisfactory, nevertheless it is possible that in some periods of time these indicators can vary significantly from their averages. In many situations these variations are highly undesirable. That is why in many cases it is important to plan the loading level of the terminal in accordance with its capacity so that the system would be able to return quickly to its normal state after the crisis situation has arisen. The considered above model allows to estimate the average time during which the terminal would be able to return to its normal stationary mode after the crisis situation has occurred which has caused the accumulation of large number of vessels in the roads. The curves of the change in the probability to find no vessels at the terminal, the change in the probability to find one vessel at the terminal, the change in the probability to find two vessels at the terminal, and the change in the probability to find all berths busy, in the case when the loading level of the terminal equals 0.27 and the road is overflowed at the beginning of the period are depicted on the picture 2.
The changes in the probability that at least one of the berths will be available if the loading level of the terminal equals 0.27 or equals 0.6 or equals 0.82, and the road is overflowed at the beginning of the period are depicted on the picture 3. The curves on picture 3 shows that although average vessels processing indicators can seem to be satisfactory, in view of the fact that the system is unable to restore quickly after the crisis situations, in some cases it would be advisable to take steps to reduce the loading level of the terminal or increase the capacity of the terminal.

Conclusions

The obtained relations between the vessels processing indicators and the loading level of the terminal may be used for the investigations of the optimal loading level which brings maximal profits for the "port-carrier-client" system. This method also can be used for the investigation of the reverse problem, that is the optimization of the various terminal characteristics in order to fit the given loading level.

References:


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